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Two-dimensional fractional supersymmetry from the quantum Poincaré group at roots of unity

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Abstract. A group theoretical understanding of the two-dimensional fractional supersymmetry is given in terms of the quantum Poincaré group at roots of unity. The fractional supersymmetry algebra and the quantum group dual to it are presented and the pseudo-unitary and irreducible representations of them are obtained. The matrix elements of these representations are explicitly constructed.

1. Introduction

Quantum algebras at roots of unity are regarded to be useful in formulations of some physical systems whose theoretical understanding is not very clear [1]. Obviously, to achieve a complete understanding of the role of these algebras in applications to physical systems one should know the quantum groups which are dual to them. Indeed, in [2] $E_q(1,1)$ at roots of unity and in [3] $SL_q(2,\mathbb{R})$ at roots of unity were constructed. In the formulation of these groups one is obliged to introduce some new variables which are the generalized Grassmannians $\eta_\pm$ satisfying $\eta_\pm^p = 0$ where p is a positive integer. On the other hand, these coordinates were used to obtain superspace realizations of the fractional supersymmetry charges [4–7]. Although some algebraic properties of the two-dimensional fractional supersymmetry were discussed in [5], the correct behaviour under the Lorentz generator could be obtained in terms of some restrictions and a spectral parameter.

A group theoretical understanding of the fractional supersymmetry appears to be lacking. Our aim is to shed some light on the group theoretical aspects of fractional supersymmetry in two dimensions. Hence, guided by the formulation of the two-dimensional quantum Poincaré algebra at roots of unity [2], we introduce the fractional supersymmetry algebra U_F endowed with a Hopf algebra structure. We present the quantum group $\mathcal{A}_F$ which is its dual. Pseudo-unitary, irreducible corepresentations of $\mathcal{A}_F$ are given and the matrix elements of them are explicitly calculated. We, then, define the pseudo-unitary quasi-regular corepresentation of $\mathcal{A}_F$ and the corresponding $*$-representation of U_F.

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2. Fractional supersymmetry algebra and its dual

Let us deal with the two-dimensional Poincaré algebra $U(e(1,1))$ generated by $P_\pm$ and H possessing the commutators

$$[P_+, P_-] = 0 \qquad [P_\pm, H] = \pm \mathrm{i} P_\pm \tag{1}$$

and the involutions

$$P_\pm^* = P_\pm \qquad H^* = H. \tag{2}$$

The two-dimensional fractional supersymmetry generators $p_\pm$ are defined to satisfy

$$p_\pm^p = P_\pm \tag{3}$$

where p is a positive integer, without any condition on the commutation relation of p_+ with p_-. Obviously, the simplest choice is $p_+p_- = p_-p_+$. Thus, the quantum Poincaré algebra at roots of unity $U_q(e(1,1))$ with $q^p = 1$, p is an odd, positive integer, generated by $p_\pm$ and κ satisfying

$$[p_+, p_-] = 0 \qquad \kappa p_\pm = q^{\pm 1} p_\pm \kappa \qquad \kappa^p = 1_U \tag{4}$$

with the involutions

$$p_\pm^* = p_\pm \qquad \kappa^* = \kappa \tag{5}$$

which suits our purposes. 1_U denotes the unit element of the algebra.

The two-dimensional fractional supersymmetry algebra denoted $U_F \equiv (U(e(1,1)), U_q(e(1,1)))$ is generated by $P_\pm$, H, $p_\pm$, κ satisfying (1)–(5) and moreover, the commutation relations

$$[\kappa, H] = 0 \qquad [p_\pm, H] = \pm \frac{\mathrm{i}}{p} p_\pm. \tag{6}$$

The latter is the consequence of (1) and (3).

The basis elements of U_F are

$$\phi^{nmkrsl} \equiv p_+^n p_-^m \kappa^k P_+^r P_-^s H^l \tag{7}$$

where n, m, k, r, s, l are positive integers.

We can equip U_F with the Hopf algebra structure

$$\begin{aligned}
&\Delta(P_\pm) = P_\pm \otimes 1_U + 1_U \otimes P_\pm \qquad \varepsilon(P_\pm) = 0 \qquad S(P_\pm) = -P_\pm \\
&\Delta(H) = H \otimes 1_U + 1_U \otimes H \qquad \varepsilon(H) = 0 \qquad S(H) = -H \\
&\Delta(p_\pm) = p_\pm \otimes \kappa + \kappa^{-1} \otimes p_\pm \qquad \varepsilon(p_\pm) = 0 \qquad S(p_\pm) = -q^{\pm 1} p_\pm \\
&\Delta(\kappa) = \kappa \otimes \kappa \qquad \varepsilon(\kappa^{\pm 1}) = 1 \qquad S(\kappa^{\pm 1}) = \kappa^{\mp 1}.
\end{aligned} \tag{8}$$

Now, we would like to present the groups which are dual to the algebras considered above.

The $*$-algebra $\mathcal{A}(E(1,1))$ of infinitely differentiable functions on the two-dimensional Poincaré group $E(1,1)$ is dual to the algebra $U(e(1,1))$. For any $f(z_+, z_-, \lambda) \in \mathcal{A}(E(1,1))$ we have the involution

$$z_\pm^* = z_\pm \qquad \lambda^* = \lambda \tag{9}$$

where

$$\begin{pmatrix} \mathrm{e}^\lambda & 0 & z_+ \\ 0 & \mathrm{e}^{-\lambda} & z_- \\ 0 & 0 & 1 \end{pmatrix} \in E(1,1).$$

The two-dimensional quantum Poincaré group at roots of unity [2] is the $*$-algebra $\mathcal{A}(E_q(1,1))$ with $q^p = 1$, generated by $\eta_\pm, \delta$ satisfying

$$\eta_-\eta_+ = q^2\eta_+\eta_- \qquad \eta_\pm\delta = q^2\delta\eta_\pm \tag{10}$$

$$\eta_\pm^p = 0 \qquad \delta^p = 1_A \tag{11}$$

$$\eta_\pm^* = \eta_\pm \qquad \delta^* = \delta \tag{12}$$

where 1_A is the unit element of $\mathcal{A}$. The dual of U_F is the $*$-algebra $\mathcal{A}_F = \mathcal{A}(E(1,1)) \times \mathcal{A}(E_q(1,1))$ with the Hopf algebra structure

$$\Delta(\eta_\pm) = \eta_\pm \otimes 1_A + \delta^{\pm1}\mathrm{e}^{\pm\lambda/p} \otimes \eta_\pm \qquad \varepsilon(\eta_\pm) = 0 \qquad S(\eta_\pm) = -\delta^{\mp1}\eta_\pm$$

$$\Delta(\delta) = \delta \otimes \delta \qquad \varepsilon(\delta^{\pm1}) = 1 \qquad S(\delta^{\pm1}) = \delta^{\mp1}$$

$$\Delta(\lambda) = \lambda \otimes 1_A + 1_A \otimes \lambda \qquad \varepsilon(\lambda) = -\lambda \qquad S(\lambda) = -\lambda$$

$$\Delta(z_\pm) = z_\pm \otimes 1_A + \mathrm{e}^{\pm\lambda}1_A \otimes z_\pm + (-1)^{\frac{p+1}{2}} \sum_{n=1}^{p-1} \frac{q^{\pm n^2}}{[p-n]![n]!}\eta_\pm^{p-n}\delta^{\pm n}\mathrm{e}^{\pm\lambda n/p} \otimes \eta_\pm^n$$

$$S(z_\pm) = -z_\pm \qquad \varepsilon(z_\pm) = 0.$$

We use the symmetric q-number

$$[n] = \frac{q^n - q^{-n}}{q - q^{-1}}$$

and the q-factorial $[n]! = [n][n-1]\ldots[1]$.

Since any function of $\mathcal{A}(E(1,1))$ can locally be expanded in Taylor series, there is a local basis of $\mathcal{A}_F$ given by

$$a^{nmktsl} \equiv \eta_+^n\eta_-^m\zeta(k,\delta)z_+^t z_-^s\lambda^l \tag{13}$$

where n, m, k, t, s, l are positive integers and we defined

$$\zeta(m,\delta) \equiv \frac{1}{p}\sum_{n=0}^{p-1} q^{-nm}\delta^n.$$

The duality relations between $\mathcal{A}_F$ and U_F are

$$\langle \phi^{nmktsl}, a^{n'm'k't's'l'} \rangle = \mathrm{i}^{n+m+t+s+l} q^{\frac{n-m}{2}-nm} l!t!s![n]![m]!\delta_{nn'}\delta_{mm'}\delta_{tt'}\delta_{ss'}\delta_{ll'}\delta_{k+n+m,k'}. \tag{14}$$

3. Pseudo-unitary, irreducible corepresentations of $\mathcal{A}_F$

Let $C_0^\infty(\mathbb{R})$ be the space of all infinitely differential functions of finite support in $\mathbb{R}$ and $P(t)$ denote the algebra of polynomials in t subject to the conditions $t^p = 1$ and $t^* = t$. The linear map

$$\pi_r(U_F) : C_0^\infty(\mathbb{R}) \times P(t) \to C_0^\infty(\mathbb{R}) \times P(t)$$

given as

$$\begin{aligned} &\pi_r(p_\pm)f(x)a(t) = (-r)^{1/p}\mathrm{e}^{\pm x/p}t^{\pm1}f(x)a(t) \\ &\pi_r(P_\pm)f(x)a(t) = -r\mathrm{e}^{\pm x}f(x)a(t) \\ &\pi_r(H)f(x)a(t) = -\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}x}f(x)a(t) \\ &\pi_r(\kappa)f(x)a(t) = f(x)a(qt) \end{aligned} \tag{15}$$

defines the irreducible representation of U_F in $C_0^\infty(\mathbb{R}) \times P(t)$.

Let us introduce the following Hermitian forms for the space $C_0^\infty(\mathbb{R}) \times P(t)$,

$$(f_1, f_2) = \int_{-\infty}^{+\infty} \mathrm{d}x\, f_1(x)\overline{f_2(x)} \tag{16}$$

$$(a_1, a_2) = \Phi(a_1(t)a_2^*(t)) \tag{17}$$

where

$$\Phi(t^s) = \delta_{s,0(\mathrm{mod}\ p)}. \tag{18}$$

$C_0^\infty(\mathbb{R})$ endowed with the norm induced by (16) leads to the Hilbert space of the square integrable functions on $\mathbb{R}$. On the other hand $P(t)$ with the pseudo-norm $\|a\|^2 \equiv (a, a)$ is the pseudo-Euclidean space with $\frac{p+1}{2}$ positive and $\frac{p-1}{2}$ negative signatures [2]. Now, one can verify that π_r defines the pseudo-unitary, irreducible $*$-representation of U_F for real r.

By making use of the duality relations (14), we can derive from (15) the irreducible corepresentations of $\mathcal{A}_F$ as

$$T_r(f(x)a(t)) = \sum_{n,m,k=0}^{p-1} \sum_{t,s,l=0}^{\infty} \frac{a^{nmktsl}\pi_r(\phi^{nmktsl})f(x)a(t)}{\langle \phi^{nmktsl}, a^{nmktsl}\rangle} \tag{19}$$

which is pseudo-unitary for real r.

Consider the Fourier transform of $f(x) \in C_0^\infty(\mathbb{R})$

$$F(\nu) = \int_{-\infty}^{+\infty} f(x)\mathrm{e}^{\nu x}\, \mathrm{d}x. \tag{20}$$

This integral converges for any complex ν. $F(\nu)$ is an analytic function and moreover, satisfies $(\nu = \nu_1 + \mathrm{i}\nu_2)$

$$|F(\nu_1 + \mathrm{i}\nu_2)| < K\mathrm{e}^{c|\nu_1|} \tag{21}$$

for some real constants K and c. Then we can write the inverse transform as

$$f(x) = \frac{1}{2\pi\mathrm{i}} \int_{c-\mathrm{i}\infty}^{c+\mathrm{i}\infty} F(\nu)\mathrm{e}^{-\nu x}\, \mathrm{d}\nu. \tag{22}$$

The Fourier transform of T_r (19) yields the pseudo-unitary corepresentation in the space of functions $F(\nu)a(t)$ as

$$Q_r(F(\nu)t^k) = \int_{c-\mathrm{i}\infty}^{c+\mathrm{i}\infty} \mathrm{d}\mu \sum_{l=0}^{p-1} Q_{kl}^r(\nu, \mu, g)F(\mu)t^l \tag{23}$$

where we denoted the variables as $g \equiv (g_0;\ g_p) = (z_+, z_-, \lambda; \eta_+, \eta_-, \delta)$ and the kernel Q_{kl}^r is for $l \geqslant k$

$$\begin{aligned} Q_{kl}^r(\nu, \mu, g) &= (q^{-1/2}\eta_+)^{l-k}\omega_{l-k}^r(\xi)\delta^k K_{l-k}^r(\nu, \mu, g_0) \\ &\quad + \omega_{p+k-l}^r(\xi)(q^{1/2}\eta_-)^{p+k-l}\delta^k K_{l-k-p}^r(\nu, \mu, g_0) \end{aligned} \tag{24}$$

for $l < k$

$$\begin{aligned} Q_{kl}^r(\nu, \mu, g) &= (q^{-1/2}\eta_+)^{p+l-k}\omega_{p+l-k}^r(\xi)\delta^k K_{p+l-k}^r(\nu, \mu, g_0) \\ &\quad + \omega_{k-l}^r(\xi)(q^{1/2}\eta_-)^{k-l}\delta^k K_{l-k}^r(\nu, \mu, g_0). \end{aligned} \tag{25}$$

We introduced, in terms of $\xi = q\eta_+\eta_-$, the polynomials

$$\omega_s^r(\xi) = \sum_{m=0}^{p-s-1} \frac{(\mathrm{i}r^{1/p})^{2m+s}}{[n]![m+s]!}(q^s\xi)^m \tag{26}$$

and the functions K_s^r are

$$K_s^r(\nu,\mu,g_0) = \frac{1}{2\pi\mathrm{i}}\mathrm{e}^{\mu\lambda}\int_{-\infty}^{+\infty}\mathrm{e}^{\mathrm{i}r(\mathrm{e}^x z_+ + \mathrm{e}^{-x}z_-)+x(\nu-\mu+s/p)}\,\mathrm{d}x. \tag{27}$$

By utilizing the analogue of polar coordinates $\rho > 0,\ \beta\in\mathbb{R}$, the pseudo-Euclidean plane defined by the axis $z_- = 0$ and $z_+ = 0$ can be studied in terms of the quadrants

$$\begin{array}{lll} \text{Quad. 1:} & z_+z_- > 0 & z_\pm = \frac{1}{2}\rho\mathrm{e}^{\pm\beta} \\ \text{Quad. 2:} & z_+z_- < 0 & z_\pm = \pm\frac{1}{2}\rho\mathrm{e}^{\pm\beta} \\ \text{Quad. 3:} & z_+z_- > 0 & z_\pm = \dfrac{-1}{2}\rho\mathrm{e}^{\pm\beta} \\ \text{Quad. 4:} & z_+z_- < 0 & z_\pm = \mp\frac{1}{2}\rho\mathrm{e}^{\pm\beta}. \end{array}$$

In these quadrants (27) will lead to the Hankel functions $H_\nu^{(1)},\ H_\nu^{(2)}$ or cylindrical functions of imaginary argument K_ν:

$$\begin{array}{ll} \text{Quad. 1:} & K_s^r(\nu,\mu,g_0) = \frac{1}{2}\mathrm{e}^{(\mu-\nu-s/p)(\beta+\frac{\pi\mathrm{i}}{2})+\mu\lambda}H^{(1)}_{\mu-\nu-s/p}(r\rho) \\ \text{Quad. 2:} & K_s^r(\nu,\mu,g_0) = \frac{1}{2}\mathrm{e}^{(\mu-\nu-s/p)(\beta-\frac{\pi\mathrm{i}}{2})+\mu\lambda}H^{(2)}_{\mu-\nu-s/p}(r\rho) \\ \text{Quad. 3:} & K_s^r(\nu,\mu,g_0) = \dfrac{1}{\pi\mathrm{i}}\mathrm{e}^{(\mu-\nu-s/p)(\beta+\frac{\pi\mathrm{i}}{2})+\mu\lambda}K_{\mu-\nu-s/p}(r\rho) \\ \text{Quad. 4:} & K_s^r(\nu,\mu,g_0) = \dfrac{1}{\pi\mathrm{i}}\mathrm{e}^{(\mu-\nu-s/p)(\beta-\frac{\pi\mathrm{i}}{2})+\mu\lambda}K_{\mu-\nu-s/p}(r\rho) \end{array}$$

with the condition $-1 < \mathrm{Re}(\nu-\mu+s/p) < 1$ [8].

4. Quasi-regular corepresentation of $\mathcal{A}_F$ and $*$-representation of the fractional supersymmetry algebra

The comultiplication

$$\Delta : \mathcal{A}\to\mathcal{A}_F\otimes\mathcal{A} \tag{28}$$

defines the pseudo-unitary left quasi-regular corepresentation of $\mathcal{A}_F$ in its subspace $\mathcal{A}$ consisting of the finite sums

$$X = \sum_s a_s(\eta_+,\eta_-)f_s(z_+,z_-)$$

where $a_s(\eta_+,\eta_-)$ are polynomials in η_+,η_- and $f_s(z_+,z_-)\in C_0^\infty(\mathbb{R}^2)$. The space $\mathcal{A}$ can be endowed with the Hermitian form

$$(X,Y) = \mathcal{I}_E(XY^*) \tag{29}$$

$X, Y\in\mathcal{A}$ and the linear functional $\mathcal{I}_E$: $\mathcal{A}\to\mathbb{C}$

$$\mathcal{I}_E(X) = \sum_s \mathcal{I}(a_s)\mathcal{I}_C(f_s) \tag{30}$$

is the left invariant integral where [2]

$$\mathcal{I}(\eta_+^n\eta_-^m) = q^{-1}\delta_{n,p-1}\delta_{m,p-1} \tag{31}$$

$$\mathcal{I}_C(f_s) = \int_{-\infty}^{+\infty}\mathrm{d}z_+\,\mathrm{d}z_-\ f_s(z_+,z_-). \tag{32}$$

The right representation of the fractional supersymmetry algebra U_F corresponding to the quasi-regular representation (28),

$$\mathcal{R}(\phi)X = (\phi\otimes\mathrm{id})\Delta(X) \tag{33}$$

$\phi \in U_F$, is a $*$-representation

$$(\mathcal{R}(\phi)X, Y)_E = (X, \mathcal{R}(\phi^*)Y)_E$$

because the Hermitian form (29) is defined in terms of the left invariant integral (30).

The right representations on the variables $\eta_\pm$ and $f(z_+, z_-)$ can explicitly be written as

$$\begin{aligned}
&\mathcal{R}(p_\pm)\eta_\pm^k = \mathrm{i}q^{\pm\frac{1}{2}}[k]\eta_\pm^{k-1} \qquad \mathcal{R}(p_\pm)\eta_\mp^k = 0\\
&\mathcal{R}(\kappa)\eta_\pm^k = q^{\pm k}\eta_\pm^k \qquad \mathcal{R}(H)\eta_\pm^n = \pm\frac{\mathrm{i}n}{p}\eta_\pm^n\\
&\mathcal{R}(p_\pm)f = \mathrm{i}q^{\pm\frac{1}{2}}\frac{(-1)^{\frac{p+1}{2}}}{[p-1]!}\eta_\pm^{p-1}\frac{\mathrm{d}f}{\mathrm{d}z_\pm} \qquad \mathcal{R}(P_\pm)f = \mathrm{i}\frac{\mathrm{d}f}{\mathrm{d}z_\pm}\\
&\mathcal{R}(\kappa)f = f \qquad \mathcal{R}(H)f = \mathrm{i}z_+\frac{\mathrm{d}f}{\mathrm{d}z_+} - \mathrm{i}z_-\frac{\mathrm{d}f}{\mathrm{d}z_-}.
\end{aligned} \tag{34}$$

In terms of the following relations satisfied by the right representation $\mathcal{R}$

$$\begin{aligned}
&\mathcal{R}(\phi\phi') = \mathcal{R}(\phi')\mathcal{R}(\phi)\\
&\mathcal{R}(p_\pm)(XY) = \mathcal{R}(p_\pm)X\mathcal{R}(\kappa)Y + \mathcal{R}(\kappa^{-1})X\mathcal{R}(p_\pm)Y\\
&\mathcal{R}(\kappa)(XY) = \mathcal{R}(\kappa)X\mathcal{R}(\kappa)Y\\
&\mathcal{R}(H)(XY) = \mathcal{R}(H)XY + X\mathcal{R}(H)Y
\end{aligned}$$

we can define the action of an arbitrary operator $\mathcal{R}(\phi)$ on any function in $\mathcal{A}$.

The quantum algebra which we deal with possesses one Casimir element $C = p_+p_-$. As the complete set of commuting operators we can choose $\mathcal{R}(C)$, $\mathcal{R}(H)$, $\mathcal{R}(\kappa)$ and $\mathcal{L}(H)$, $\mathcal{L}(\kappa)$ where $\mathcal{L}(\phi)$ is the left representation of the element ϕ defined similar to (33) with the interchange of ϕ with the identity id. One can easily observe that $\mathcal{L}(H)X = 0$ and $\mathcal{L}(\kappa)X = X$ for any $X \in \mathcal{A}$, so that, in the space $\mathcal{A}$ the matrix elements can be labelled as $D^r_{n\nu,m\mu}$. Indeed, in terms of the kernel Q^r_{mn} (24) one observes that

$$D_{n\nu,00} = Q^r_{0n}(\nu, 0, g)$$

$n \in [0, p-1]$, satisfy

$$\begin{aligned}
&\mathcal{R}(\kappa)D_{n\nu,00} = q^n D_{n\nu,00}\\
&\mathcal{R}(H)D_{n\nu,00} = -\mathrm{i}(\nu + n/p)D_{n\nu,00}\\
&\mathcal{R}(C)D_{n\nu,00} = c^2 D_{n\nu,00}\\
&\mathcal{R}(p_+)D_{n\nu,00} = cD_{n+1\nu,00}\\
&\mathcal{R}(p_-)D_{n\nu,00} = cD_{n-1\nu,00}\\
&\mathcal{R}(P_\pm)D_{k\nu,00} = -rD_{k\nu\pm1,00}
\end{aligned}$$

where $c = r^{1/p}q$ and we introduced the notation $D_{p\nu,00} \equiv D_{0\nu+1,00}$ and $D_{-1\nu,00} \equiv D_{p-1\nu-1,00}$.

The right representation obtained in (34) can be used to write the supercharge operators $\mathcal{R}(p_\pm)$ in the superspace given by η_+, z_+ or η_-, z_- as

$$\mathcal{R}(p_\pm) = \mathrm{i}q^{\pm\frac{1}{2}}D^q_\pm + \frac{(-1)^{\frac{p+1}{2}}}{[p-1]!}\eta_\pm^{p-1}\frac{\mathrm{d}}{\mathrm{d}z_\pm} \tag{35}$$

where $D^q_\pm$ are q-derivatives with respect to $\eta_\pm$. This is the same with the realization of supercharges given in [4–7], obtained in terms of q-calculus.

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